Personal Bankruptcy Law and Small Firms Finance: Extensive and Intensive Margin

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Abstract

It is an empirical fact that there exists a non-monotonic relation between bankruptcy exemptions and small firms' loan size as well as the interest rates (Berkowitz and White, 2004). Up to now, the literature has not been successful at accounting for this relationship. In this paper, I present a principal-agent model with banks' screening that replicates the non-monotonic relation between bankruptcy exemptions and small firms' loan size as well as the interest rates, as observed in the data. Furthermore, this paper provides new insights into the relationship between bankruptcy exemptions and aggregate production. In particular, too high or too low bankruptcy exemptions lead to a decrease of aggregate production as the exemption increases since banks grant the loan to all firms without screening. A mid-way value of bankruptcy exemption results in an increase of aggregate production as the exemption increases. This result comes from introducing the *extensive margin* by allowing banks' screening. I conclude with a discussion of potential testable implications from the model.

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1 Introduction

Small businesses are the primary source of new jobs in the U.S. economy. Data published by the U.S. Census Bureau clearly show that, from 1990 to 2003, small firms (businesses with less than 20 employees) accounted for 79.5% of net new jobs, i.e., the net of job creation and job destruction, both weighted by the firm-level employment growth rate. Moreover, small business are considered to play an important role in economic growth (Acs and Audretsch (1990), Cohen and Klepper (1996), Wennekers and Thurik (1999), Audretsch (2002), Edmisten (2007)). This evidence stresses the important role of small businesses in the U.S. economy. However, small business entrepreneurs have fewer assets and depend primarily on debt financing. Specifically, 50.37% of small businesses' external finance is debt finance among all sources of finance. Commercial banks contribute 18.75% of debt finance (Berger and Udell (1998)). Since obtaining credits from banks is important for small business to invest in projects, the factors that affect small business' access to credit in the credit market play a crucial role. One of these factors is the personal bankruptcy law.

In the U.S., it is well known that individual and corporate bankruptcy procedures are separated. Moreover, the U.S. personal bankruptcy law not only applies to consumers, but also to non-corporate small businesses. The reason is that, if a firm is non-corporate, its debt is the personal liability of the owner of the firm. The objective of the personal bankruptcy law is to offer a security to debtors when they are in financial distress, for instance, a negative income shock, job loss, or health problem. Borrowers can choose between filing Chapter 7 and Chapter 13.

Under Chapter 7, borrowers' unsecured debts are discharged and they repay and give up all their assets beyond a certain level which is predetermined by the bankruptcy exemption. Also, after filing Chapter 7, they do not have any obligation of repaying the debt in the future. In other words, their future earnings are completely exempt. In the U.S., the bankruptcy law is a federal law and the procedure is uniform across the country. However, each state has its own right to set the exemption level. Most states have several types of exemptions, such as homestead (the borrowers' own resident house) exemption, equity in vehicles and in other goods, etc.. In most of the states, the homestead exemption is the largest one. Table 1 presents the homestead exemption level across states in the US in 1993.

The link between bankruptcy law and firms' finance has been examined in a great amount of theoretical and empirical works. In particular, in a series of works by Berkowitz and White (2000, 2004), Gropp *et al.* (1997), and LaPorta *et al.* (1998), the impact of personal bankruptcy law on small business finance is emphasized. This literature leads to a result that a higher protection of creditors' right would give banks incentives to provide more abundant and cheaper credit to the entrepreneurs (*intensive margin*). This literature has focused on the *intensive margin*, specifically, on the effects of bankruptcy exemptions on firms' loan. LaPorta *et al.* (1998) point out that a lower creditor protection, through higher bankruptcy exemptions, affects banks' incentives to provide (full) credit to the entrepreneurs. Gropp *et al.* (1997) analyze how cross-state differences in U.S. personal bankruptcy rules affect the supply and demand for household credit, using data from the 1983 Survey of Consumer Finances. They find that generous state bankruptcy exemptions reduce the amount of credit available to low-asset households and increase the interest rates on automobile loans. This literature might lead to the conclusion that the lower exemption leads to more abundant and cheaper credit. Thus, it might be concluded that in order to improve the efficiency of the credit market, a low (or even zero) exemption is desired. However, in a later work, Berkowitz and White (2004) provide evidence that there is a non-monotonic relation between the exemption levels and the loan size, the probability of credit rationing and the interest rate, which contrasts with the previous literature, in which a monotonic negative relation is derived.

Table 1: 1993 Bankruptcy Exemptions by States

(source: Berkowitz and White (2004))

Homestead Exemption ($)		Homestead Exemption ($)	
Alabama	10,000	Montana	80,000
Alaska	54,000	North Carolina	20,000
Arizona	100,000	North Dakota	160,000
Arkansas	Unlimited	Nebraska	20,000
California	75,000	New Hampshire	60,000
Colorado	60,000	New Jersey	15,000
Connecticut	15,000	New Mexico	40,000
D.C	15,000	Nevada	95,000
Delaware	15,000	New York	20,000
Florida	Unlimited	Ohio	10,000
Georgia	10,000	Oklahoma	Unlimited
Hawaii	40,000	Oregon	20,000
Iowa	Unlimited	Pennsylvania	15,000
Idaho	100,000	Rhode Island	15,000
Illinois	15,000	South Carolina	15,000
Indiana	15,000	South Dakota	60,000
Kansas	Unlimited	Tennessee	7,500
Kentucky	10,000	Texas	Unlimited
Louisiana	15,000	Utah	10,000
Massachusetts	100,000	Virginia	10,000
Maryland	0	Vermont	60,000
Maine	15,000	Washington	60,000
Michigan	15,000	Wisconsin	40,000
Minnesota	Unlimited	West Virginia	15,000
Missouri	8,000	Wyoming	20,000
Mississippi	150,000		

Previous literature has neglected the fact that investment projects undertaken by firms that obtained credit are heterogeneous in terms of the quality. Without this heterogeneity, it is impossible to explain the non-monotonic relation. In this paper, I relax this assumption that all investment projects are of the same quality. I argue that in order to make sure the credits to be used productively, banks would screen the projects undertaken by their loan applicants. Since a good-quality project is more likely to be successful compared with a bad-quality project, banks would prefer only funding the good-quality projects. As a result, screening by banks is socially desirable. It increases aggregate production in terms of both *intensive* and *extensive margin*, since banks do not provide funds to all firms, but only to those firms with good-quality investment opportunities (*extensive margin*). In addition, the firms with good-quality projects obtain more credit (*intensive margin*). The banks' incentives closely depend on the level of the protection of the creditors' right. The banks' incentive to screen is low when their rights of possessing the firms' assets are better protected. In other words, if the exemption level is low, banks may lack incentives to screen. As a result, my model is able to replicate the non-monotonic relation between bankruptcy exemptions and small firms' access to credit by allowing investment projects of different quality and introducing banks' screening.

Manove, Padilla and Pagano (MPP 2001), and Garmaise (2001) argue that banks have the ability to screen projects and hence could distinguish good-quality projects from bad-quality ones. Banks often fund a large number of investment projects in some specific sectors. Hence, they have considerable experience dealing with similar projects undertaken by firms in these sectors. Moreover, they have more information about the general economic trend. This enables banks to know the quality of the projects chosen by entrepreneurs better than entrepreneurs themselves. Banks' incentives to screen depend on the levels of creditors' protection. MPP suggest that the screening decision of the creditors is negatively related to the level of creditors' protection, and also negatively depends on the amount of collateral that firms provide. They conclude that if the creditors' protection is low, high-quality firms would offer a high collateral to distinguish themselves from low-quality ones.

This paper aims at analyzing not only how the banks' screening decisions along with different levels of bankruptcy exemptions affect small firms' access to credits (*intensive margin*), but also how this screening affects the quality of the set of funded projects (*extensive margin*).

I develop a principal-agent model of competitive banking in the credit market. In my model, investment projects are of different quality. Banks, which are the only lenders, are able to learn the quality of a project undertaken by a firm by exercising a costly screening. The model shares the same insight with MPP's paper which endogenizes the screening decision of banks. However, in their framework, they do not analyze the relationship between banks' screening intensity with different bankruptcy exemptions. Moreover, the loan size as capital investment is fixed in their environment, and thus, the implication of lending on the *intensive margin* cannot be analyzed. Here, on the other hand, I allow firm' capital investment to be endogenous. This allows us first to replicate the non-monotonic effect of the bankruptcy exemptions on the loan size, and hence to analyze the relation between bankruptcy exemptions and aggregate production. Our result suggests that bankruptcy exemptions have a non-monotonic effect on banks' optimal screening intensity, and moreover, the banks' screening intensity positively affects the optimal loan size. Hence, bankruptcy exemptions and optimal loan size display a non-monotonic relation, as observed in the data. The model further implies a non-monotonic relation between bankruptcy exemptions and aggregate production. Due to banks' screening, the probability of a firm's success is higher. On the one hand, banks' screening distinguishes good-quality projects from bad-quality ones, although not perfectly. Banks screen out the bad-quality projects and fund good-quality projects. As a result, the set of funded projects contains a higher proportion of good-quality projects. This is the *extensive margin* coming from banks' screening. On the other hand, given that the set of funded projects contains higher proportion of good-quality projects, banks are willing to lend more to every funded firm, and this in turns increases firms' probability of success since firms' capital investment increases. This is the *intensive margin* coming from the larger capital investment made by firms. I find that when exemptions are neither too high nor too low, banks screen and the screening intensity is increasing as the exemption rises. As a consequence, the relation between bankruptcy exemptions and aggregate production is non-monotonic and hence, a higher bankruptcy exemption does not necessarily lead to a lower aggregate production. This non-monotonicity arises not only from the non-monotonic behavior of loan size and exemptions, but also from the change of the proportion of funded projects being of good quality due to banks screening. I further embed the analysis in an environment that potential entrepreneurs display heterogeneous risk attitudes. A low bankruptcy exemption discourage risk-averse individuals for becoming entrepreneurs. Finally, I conjecture that a mid value exemption would lead to a highest social welfare.

This paper contributes to the literature on the importance of bankruptcy law on small firms' access to credits by combining the *intensive* and the *extensive margin* of banks' lending in the analysis. I show that the gain of a firm's expected production, when the bankruptcy exemption is middling, comes from two channels. On the one hand, banks screen out the bad-quality projects, although imperfectly, and fund good-quality projects; on the other hand, banks also provide more loans as capital investment to funded firms. This result suggests that it would be socially desirable to regulate the bankruptcy exemption at this middling range. Besides, an improvement in screening technology would leads to a larger gain from both *extensive* and *intensive margin*. This provides testable implications for future research.

The rest of the paper is organized as follows: Section 2 describes the setup of the model. In Section 3, I derive the equilibrium debt contracts under perfect competition and analyze the effect of bankruptcy exemptions on the equilibrium contracts. Section 4 analyzes the effect of bankruptcy exemptions on firms' aggregate production and demonstrates that banks' screening improves the allocated efficiency in the credit market. Section 5 further discusses how the relation between bankruptcy exemptions and the equilibrium loan size changes if the screening technology is improved. Finally, I conclude in Section 6.

2 The Setup

2.1 Agents and environment

Consider a one-period principal-agent model in the credit market. Firms (agents) are endowed with homestead assets which value A in the market[1]. Each firm has an opportunity to undertake an investment project. Firms' utility functions are quasi-linear. In particular, firms' utility is concave in their homestead assets. Specifically, I assume that the utility of their homestead is $V(A)$ where $V(.) \geq 0$, $V'(.) > 0$ and $V''(.) < 0$. Prior to undertaking projects, firms choose their homestead assets A^* so that $V'(A^*) = 1$ and $V(A^*) > A^*$. As a result, firms will not liquidate their homestead assets to invest in the projects since the firms value their homestead assets higher than the market value. Therefore, firms have to obtain the funds from risk neutral banks (principals). This could be implied by the fact that homestead assets are not perfectly liquid. In practice, once the firms file for bankruptcy and their homesteads are liquidated, some fixed cost of liquidation, such as the commission to the trustee, would be borne. Therefore, the money received by the creditors' from liquidation must be lower than the value of the assets.

The investment projects are of two qualities: good (g) or bad (b). A good-quality project yields an observable random return z with two possible outcomes,

$$z = \begin{cases} \theta & \text{if the project succeeds} \quad \text{with probability } p(B) \\ 0 & \text{if the project fails} \quad \text{with probability } 1 - p(B) \end{cases}$$

Further, without loss of generality, I assume a bad-quality project always yields return 0. This assumption does not affect my analysis as long as the expected outcome of a good-quality of project is higher than the expected outcome of a bad-quality project. The distribution of the realized returns of a good-quality project is endogenous, and it depends on the funds B invested in the project. The probability of success of a good-quality project is denoted as $p(B)$, with $p'(B) > 0$, $p''(B) \leq 0$ for all B.

[1] Here I assume that A is homestead asset. However, we could also consider A includes homestead, cash, and other assets and our results still hold.

The proportion of good-quality projects among all projects q is exogenous. Without loss of generality, I assume the proportion $q = \frac{1}{2}$, and it is known to both firms and banks. Moreover, firms cannot distinguish the quality of the project they have chosen[2]. However, banks can screen and thus obtain a (imperfect) report about the quality of the projects. The report from screening takes two outcomes: $R = \{Rg, Rb\}$ .The accuracy of banks screening depends on the exerted screening intensity s by banks. The screening intensity s is private information and not observable to the firms and hence not contractible. I denote $\Pr(Rg|g,s)$ as the probability that a bank can recognize a good-quality project by exerting s, and $\Pr(Rg|b,s)$ as the probability that a bank mis-believes a bad-quality project as a good one. In particular, the probabilities are written as follows:

$$\Pr(Rg|g,s) = 1$$

$$\Pr(Rg|b,s) = \pi(s) = e^{-s}$$

$\Pr(Rg|b,s)$ is decreasing in s, and $\pi(s) = 1$ for $s = 0$, $\pi'(s) \leq 0$, $\pi''(s) > 0$ for $s \in (0,\infty)$. The above assumption of conditional probabilities does not affect the analysis, even if $\Pr(Rg|g,s)$ depends on s as well. What is needed in the analysis is that screening can improve the knowledge of banks about the project quality. For simplicity, I assume $\Pr(Rg|g,s) = 1$. Screening is costly, and the cost of screening is defined as $C(s) = c_f + cs$, where c_f is a quasi-fixed cost and cs is a variable cost.

Finally, $(B, r|A^*)$ represents the standard debt contract with the loan size B and interest rate r for a given A^*. Both B and r are observable and enforcable. Once a contract is adopted, a firm can fully commit to obtain the loan from the associated bank if the loan application is approved. Since the contract is enforceable, the firm does not have incentives to deviate. In other words, if the report is good, $R = Rg$, the firm cannot go to another bank. Enforceability of contracts is crucial since if the contract is not enforceable,

[2] This assumption could be relaxed without changing the results in the paper. In particular, we could assume that both firms and banks have their own beliefs about the exogenenous proportion of good-quality projects. Their own beliefs need not to coincide nor to be equal to the exogenous proportion q.

a firm would have incentive to go to other banks when the report is good. In equilibrium, no banks would screen and the firm is worse off if there is no screening. Thus, enforceability is in firms' interest as well. If the report is bad, $R = Rb$, the firm would not go to other banks since the probaility of having a bad-quality project is larger than the unconditional probability $1 - q$. The probability of a project being of bad quality given rejected is equal to 1. As a result, in equilibrium, a firm only goes to one bank.

2.2 Personal Bankruptcy law

The realized returns of the project are observable and verifiable. When a project yields return 0, firms are not able to repay their debt and thus, they file for bankruptcy. Under Chapter 7, once a firm files for bankruptcy, its debt is exempted and its assets are liquidated up to the exemption level E. In other words, the bank partially liquidates the firm's initial assets $(A - E)$, and the firm keeps the other part of the asset E, which implies a utility $V(E)$[3]. Different levels of bankruptcy exemptions indicate different degrees of the protection of creditor right. In particular, a higher level of bankruptcy exemptions implies a lower degree of creditors' protection.

2.3 Timeline and Competitive Equilibrium

The relationship between firms and banks occurs in the following way:
(1) Firms randomly pick a project, and the quality of the project is unknown to them.
(2) Given E, banks determine their screening intensity and post debt contracts $(B, r|A^*)$. Banks compete with each other by taking the debt contracts offered by other banks as given.

[3]In practice, different states have value-based homestead exemption or lot-size-based homestead exemption. In the case of lot-size-based exemption, part of the homestead in term of the size is liquidated. In the case of value-based exemption, the homestead is entirely liquidated, and the individual could keep the value of the homestead which is equal to the exemption. This assumption could be interpreted as the lot-sized based exemption or the individual can get a less valuable house after the original homestead is liquidated.

(3) Each firm applies for at most one contract. In other words, if the application for a loan is rejected, firms do not invest[4].
(4) Each bank decides whether to approve the loan applications given the received report about the project quality.
(5) Funded project returns are realized. The firms repay their debt, Br. In the case of failure to repay their debt, the firms file for bankruptcy and banks liquidate the firms' assets for recovering their credits.

Under perfect competition, the equilibrium debt contracts maximize a firm's expected utility subject to a bank's zero-profit condition. If a bank does not screen projects at all, the firm is funded since the bank cannot distinguish a good-quality project from a bad-quality project. If the bank screens, the funding decision of the bank depends on the received report after screening:

$$R = \begin{cases} Rg, & \text{Bank funds} \\ Rb, & \text{Bank rejects} \end{cases}$$

After screening, a bank's funding decision depending on the report of screening. If the report $R = Rb$, the bank would not fund the project because the bank believes that this project would end up with return zero. If the report is $R = Rg$, the bank would fund the project since screening is informative but costly. In other words, assume the bank does not fund the project, the bank would simply lose the screening cost. Hence, this would not happen in equilibrium.

In summary, a competitive equilibrium in the credit market is characterized by:
(a) a debt contract for given A^*, $(B^*, r^*|A^*)$
(b) a screening intensity $s^* \in [0, \infty)$ exerted by banks
(c) a funding rule: funding a project if the report is good; rejecting a project if the report is bad.

[4] See the argument in Section 2.1.

3 A competitive credit market equilibrium

The funding rule states that banks only fund the projects if the received report indicates that the projects are good. Besides, since the screening intensity is unobservable to firms, the funded firms' expected utility is

$$\frac{1}{2}\left[p\left(B\right)\left(\theta - Br + V\left(A^*\right)\right) + \left(1 - p\left(B\right)\right)V\left(E\right)\right] + \frac{1}{2}V\left(E\right)$$

The equilibrium contracts maximize the funded firms' expected profit subject to banks' participation constraints. Therefore, an equilibrium contract is the solution to the following maximization problem:

$$\max_{B,r,s} EU = \frac{1}{2}\left[p\left(B\right)\left(\theta - Br + V\left(A^*\right)\right) + \left(1 - p\left(B\right)\right)V\left(E\right)\right] + \frac{1}{2}V\left(E\right)$$

s.t.

$$E\pi = \frac{1}{2}\left[p\left(B\right)Br + \left(1 - p\left(B\right)\right)\left(A^* - E\right) - B\right] + \frac{1}{2}\pi\left(s\right)\left(A^* - E - B\right) - C\left(s\right) \geq 0 \qquad (\mathrm{PC}_B)$$

$$EU = \frac{1}{2}\left[p\left(B\right)\left(\theta - Br + V\left(A^*\right)\right) + \left(1 - p\left(B\right)\right)V\left(E\right)\right] + \frac{1}{2}V\left(E\right) \geq V\left(A^*\right) \qquad (\mathrm{PC}_F)$$

Lemma 1 shows that under perfect competition, each bank's participation constraint binds in equilibrium.

Lemma 1 $E\pi^* = 0$

This result can be shown by contradiction. From the binding participation constraints of banks, the equilibrium gross interest rate is

$$r^* = \frac{2C\left(s\right) + \pi\left(s\right)\left(B - A^* + E\right) - \left(1 - p\left(B\right)\right)\left(A^* - E\right) + B}{p\left(B\right)B}$$

Under perfect competition, the equilibrium screening intensity s^* minimizes the gross interest rate for any loan size B subject to firms' participation constraints and a given bankruptcy exemption E.

Proposition 2 *(Equilibrium screening intensity)*

The equilibrium screening intensity s^* *satisfies*

(1) $s^* = 0$ *if* $E \in [0, \underline{E}] \cup \left[\overline{E}, \infty\right)$.

(2) $s^* > 0$ *if* $E \in \left(\underline{E}, \overline{E}\right)$, *where* s^* *satisfies* $\pi'(s^*) = \frac{-2c}{B^* - A^* + E}$.

Proposition 2 shows that if the bankruptcy exemption is low or high, in equilibrium, banks will not screen the firms' projects at all. However, if the bankruptcy exemption is at middle levels, banks will screen the firms' projects at an intensity which positively relates with bankruptcy exemptions.

Whether banks screen or not depends on whether the benefits from screening can cover the cost of screening. If the bankruptcy exemption is low, even if the firms default and file for bankruptcy, banks still can recover most of the credits by liquidating the firms' assets. Therefore, the benefit from screening is small. However, banks have to spend a cost $C(s) = c_f + cs$ if they screen. If the quasi-fixed cost c_f is not too small, it is not worthy for the banks to screen the firms' projects. If the bankruptcy exemption is very high, due to the convexity of $\pi(s)$, banks' screening intensity should be very high if the banks screen. In turn, banks have to considerably increase the gross interest rate in order to break even. Therefore, if banks screen, in equilibrium, firms would not apply for the loan since the cost of credits (gross interest rate) is too high. Hence, the firms would not undertake the investment projects. As a result, banks do not screen when the bankruptcy exemption is very high.

Banks' screening decisions are endogenous. Consequently, the equilibrium loan size B not only depends on the exogenous bankruptcy exemptions, but also on the banks' screening decisions (which depends on bankruptcy exemptions). In the following analysis, I disentangle the total effect of bankruptcy exemption on the equilibrium loan size into two effects: a *direct effect* and an *indirect effect*.

The *direct effect* is the effect of bankruptcy exemption directly on the equilibrium loan size given a fixed banks' screening intensity. The following lemma states this result formally.

Lemma 3 *(direct effect on* B*)*

Given a fixed $s = s'$, $\frac{\partial B^*}{\partial E}|_{s=s'} < 0$ *for any* E

Lemma 3 shows that the *direct effect* is always negative. An increase in bankruptcy exemptions leads to a decrease in the equilibrium loan size given a fixed screening intensity of banks. The intuition of this result is that, an increase in bankruptcy exemptions increase the firms' utility if the firms go bankrupt. On the other hand, if the bankruptcy exemption increases, banks would raise the gross interest rate in order to break even. This in turn decreases the firms' utility outside of bankruptcy. Consequently, the marginal utility of loan size decreases as the bankruptcy exemption increases. Therefore, an increase in bankruptcy exemptions leads to a decrease in the equilibrium loan size if banks' screening intensity is constant.

However, the equilibrium loan size also depends on the banks' screening intensity which is determined by the bankruptcy exemptions. This is the *indirect effect.*

Lemma 4 *(indirect effect on* B*)*
$\frac{\partial B^*}{\partial s^*}\frac{\partial s^*}{\partial E} > 0$ *for all* E

Lemma 4 shows that the *indirect effect* is positive. An increase in bankruptcy exemptions increases the banks' screening intensity and thus leads to an increase in the equilibrium loan size. The intuition of the result is the following: if the bankruptcy exemption increases, the loss of the banks becomes larger if the firms file for bankruptcy. Hence, under perfect competition, in order to maintain a zero-profit condition, banks increase the screening intensity. As a result, in order to increase the marginal utility of the loan, the equilibrium loan size increases.

From Proposition 2, Lemma 3 and Lemma 4, if bankruptcy exemptions are low or high, an increase in bankruptcy exemptions certainly leads to a decrease in the equilibrium loan size.

Proposition 5 *If* $E \in [0, \underline{E}] \cup \left[\overline{E}, \infty\right)$*, an increase in bankruptcy exemption* E *leads to a decrease in the equilibrium loan size* B^*

Proposition 5 is straightforward. Since if the bankruptcy exemption is very low ($E \in [0, \underline{E}]$) or very high $\left(E \in \left[\overline{E}, \infty\right)\right)$, banks do not screen any projects and thus $s^* = 0$. Therefore, there is no *indirect effect* of bankruptcy exemptions on equilibrium loan size. The total effect comes from the *direct effect*. Following Lemma 3, the *direct effect* of bankruptcy exemptions on the equilibrium loan size is negative.

However, if the bankruptcy exemption is in the middle level, the *direct* and *indirect effects* coexist. Since they have opposite effects on the equilibrium loan size, the total effect is ambiguous. Consequently, the result depends on the magnitudes of the two effects. In the following proposition, I show that if the bankruptcy exemption is in the smaller partition within the middle level (i.e., $E \in [\underline{E}, E_1]$), the *direct effect* dominates the *indirect effect*. As the bankruptcy exemption continues increasing, in particular, if the bankruptcy exemption $E \in \left(E_1, \overline{E}\right)$, the *indirect effect* dominates the *direct effect*.

Proposition 6 *If the equilibrium screening intensity $s^* > 0$, there exists a threshold E_1 so that an increase in bankruptcy exemptions leads to*

(1) a decrease in equilibrium loan size B^* if $E \in [\underline{E}, E_1]$

(2) an increase in equilibrium loan size B^* if $E \in \left(E_1, \overline{E}\right)$

Proposition 6 demonstrates that bankruptcy exemptions affect the magnitudes of the two effects. The result is driven by the concavity of the function $V(.)$ as well as the convexity of $\pi(s)$. When the exemption is in the lower end of a middle level, an increase in the bankruptcy exemption leads to a large increase in the firms' utility at bankruptcy. Besides, the firms' utility outside of bankruptcy decreases considerably as well. As a result, the equilibrium loan size decreases since the marginal utility of the loan is small. On the other hand, when the bankruptcy exemption keeps rising, the increase of the firms' marginal utility at bankruptcy, as well as the decrease of the firms' marginal utility outside of bankruptcy, become smaller. This in turn increases the marginal utility of the loan when the bankruptcy exemption gets larger ($E \in \left(E_1, \overline{E}\right)$). Consequently, the equilibrium loan size increases.

Finally, I combine the results from Proposition 5 and 6, and I derive the relation between bankruptcy exemptions and the equilibrium loan size. This result is shown in the following proposition.

Proposition 7 *(Total effect of bankruptcy exemptions on the equilibrium loan size)*

An increase in bankruptcy exemptions E (when $E \in [0, E_1]$) first leads to a decrease in the equilibrium loan size B^*. If the exemption continues increasing ($E \in \left[E_1, \overline{E}\right]$), the equilibrium loan size B^* increases as E increases, but finally, if the exemption continues rising ($E \in \left[\overline{E}, \infty\right)$), the equilibrium loan size B^* decreases as the bankruptcy exemption increases further.

The result of Proposition 7 is simply a consequence of combining the two results of Proposition 5 and 6.

As for the effect of bankruptcy on the equilibrium gross interest rate r^*, in the following proposition, I show that there is also a non-monotonic relation between bankruptcy exemptions and the equilibrium gross interest rate. In particular, the effect on the equilibrium gross interest rate is opposite to the effect of bankruptcy exemptions on the equilibrium loan size in Proposition 7.

Proposition 8 *(Total effect of bankruptcy exemptions on equilibrium gross interest rate)*

An increase in bankruptcy exemptions E (when $E \in [0, E_1]$) first leads to an increase in the equilibrium gross interest rate r^*. If the exemption continues increasing ($E \in \left(E_1, \overline{E}\right)$), the equilibrium gross interest rate r^* decreases as E increases, but finally, if the exemption continues rising ($E \in \left[\overline{E}, \infty\right)$), the equilibrium gross interest rate r^* increases as the bankruptcy exemption increases further.

I have analyzed the relation between bankruptcy exemptions and the equilibrium debt contracts by emphasizing the importance of the banks' role of screening beside of providing funds to the firms. Our model yields a non-monotonic relation between bankruptcy exemptions and the equilibrium loan size as well as the equilibrium gross interest rate, which is consistent with the data (Berkowitz and White (2004)).

In the following sections, I further show that our model is not only able to replicate the non-monotonic relation as observed in the data, but also offers further testable implications. In particular, I first analyze the relationship between bankruptcy exemptions and aggregate production, and show that this relation is non-monotonic as well. This suggests that due to banks' screening, banks are able to distinguish different qualities of the projects and fund a larger proportion of good-quality projects. This increases a funded firm's expected production. It also implies that for a given exemption, an economy with banks' screening has a higher proportion of good-quality projects among all the funded projects than one without banks' screening. Furthermore, in Section 5, I predict the relation between bankruptcy exemptions and equilibrium loan size if the screening technology is improved.

4 Bankruptcy Exemptions and Aggregate Production

In this section, I analyze how bankruptcy exemptions affect aggregate production. In particular, first I derive the expected production of a funded firm as a function of the bankruptcy exemption.

A firm's expected production is the probability of success of the firm times the positive return θ. The firm's probability of success depends on both the probability of a chosen project being of good quality and the probability of success once the good-quality project is chosen:

$$\text{Expected production} = \frac{q}{q + (1-q)\,\pi(s)} p\left(B(s)\right)\theta$$

Following the analysis in Section 3, the probability of a funded project being of good quality is $\frac{q}{q+(1-q)\pi(s)}$, which depends on banks' screening intensity s. Since $\pi(s)$ is a decreasing function of s, $\frac{q}{q+(1-q)\pi(s)}$ increases as the screening intensity s increases.

As shown in Proposition 2 and 5, when the bankruptcy exemption E is lower than the threshold $\underline{E}$ or is higher than the threshold $\overline{E}$, banks do not screen. In this case, all firms obtain credit. The probability of a funded project being of good quality is equivalent to the unconditional probability of a project being of good quality q and is exogenous. Following the setup in Section 2, this probability is assumed $q = \frac{1}{2}$. Hence, in this case, a firm's expected production is $\frac{1}{2}p(B)\theta$, which depends only on the capital investment B. Following from Lemma 3, without banks' screening, as the bankruptcy exemption increases, the capital B decreases. As a result, the firm's expected production $\frac{1}{2}p(B)\theta$ is decreasing as the bankruptcy exemption increases since banks do not screen.

However, banks do not just provide credits. By screening, banks have the ability to distinguish, although not perfectly, good-quality projects from bad-quality ones. Banks' incentives to screen depend crucially on a given bankruptcy exemption. In particular, as shown in Proposition 2, when $E \in \left[\underline{E}, \overline{E}\right]$, banks increase the screening intensity as the bankruptcy exemption increases. This enables banks to more precisely distinguish good-quality projects from bad-quality ones. In particular, a firm with a good-quality project would always obtain credit. However, some of the bad-quality projects may be misbelieved to be good-quality ones since screening is not perfect. Therefore, when $E \in \left[\underline{E}, \overline{E}\right]$, as the bankruptcy exemption increases, the probability of a funded project being of good quality $\frac{1}{1+\pi(s)}$ increases as well. The relation between a funded firm's expected production and bankruptcy exemptions is derived in the following proposition.

Proposition 9 *There exists a threshold $E_2 \in (\underline{E}, E_1)$,*

(1) If $E \in [\underline{E}, E_2]$, the expected production $\frac{1}{1+\pi(s)}p(B)\theta$ decreases as the bankruptcy exemption E increases.

(2) If $E \in \left[E_2, \overline{E}\right]$, the expected production $\frac{1}{1+\pi(s)}p(B)\theta$ increases as the bankruptcy exemption E increases.

In Section 3, Proposition 6 shows that there is a threshold E_1, where $\underline{E} < E_1 < \overline{E}$. When $E \in \left[E_1, \overline{E}\right]$, the loan size B increases as the bankruptcy exemption increases. As a consequence, the expected production $\frac{1}{1+\pi(s)}p(B)\theta$

increases for certain. When $E \in [\underline{E}, E_1]$, the loan size B decreases as the bankruptcy exemption increases. Proposition 9 shows that for $E \in [E_2, E_1]$, even though $p(B)$ decreases as the bankruptcy exemption increases, the overall effect of bankruptcy exemptions on the expected production is still positive. This result implies that given an optimal screening intensity $s^* > 0$, a firm's expected production is maximized at $E = \overline{E}$. Overall, the model predicts that the relation between bankruptcy exemptions and a firm's expected production is non-monotonic. This non-monotonicity arises from the combination of the *intensive* and *extensive margin*. This shows that a higher exemption does not necessarily lead to a lower expected production due to the *extensive margin*. However, the optimal level of exemption for achieving a highest expected production depends on the functional form of $p(B)$, as well as the parameters.

As for the aggregated effect of bankruptcy exemptions on expected production of funded firms, it follows the same analysis as in the single firm case. For heterogeneous entrepreneurs with different initial homestead assets A^*, the non-monotonic relation remains for each entrepreneur, but with different thresholds depending on A^*. As a result, the relation between bankruptcy exemptions and aggregate production is still non-monotonic.

Our model predicts that the non-monotonic relation between bankruptcy exemptions and aggregate production arises from combining both the *intensive* and *extensive margin* in the analysis. In particular, when the bankruptcy exemption takes a mid value, the aggregate production is not necessarily lower than the aggregate production when the exemption is very low. The reason is that a mid-value exemption induces banks screening. By screening, banks are able to distinguish the quality of the projects, and thus the proportion of good-quality projects being funded is higher (*extensive margin*). Moreover, given the fact that a funded project is more likely to be of good quality, credit flows from bad-quality projects to good-quality projects. Therefore, funded projects obtain more credit (*intensive margin*).

5 Discussion: An Improvement in Screening Technology

In this section, I discuss the effect of bankruptcy exemptions on the equilibrium loan size if the banks' screening technology has been improved. I consider three different types of improvements: two of them are different reductions in the cost of screening and the other is an improvement in the accuracy of screening.

First, suppose that the quasi-fixed screening cost c_f is reduced. Banks would already start screening when the bankruptcy exemption is very low. In other words, the threshold $\underline{E}$ would decrease. Proposition 2 shows that the quasi-fixed cost c_f determines at which exemption level banks start screening. After a reduction in c_f, the new threshold $\underline{E}'$ will be smaller than the old threshold $\underline{E}$. However, the reduction of the quasi-fixed cost c_f does not affect the threshold $\overline{E}$ nor E_1, since both are independent of c_f. Therefore, if c_f reduces sufficiently, the new threshold $\underline{E}'$ would go close to *zero*. As a consequence, the effect of bankruptcy exemptions on the equilibrium debt contract is still non-monotonic as found in Berkowitz and White (2004) since E_1 remains unchanged.

Second, if the per-unit cost of screening decreases, i.e., c decreases to c' ($c' < c$), the thresholds $\underline{E}$, $\overline{E}$ and E_1 all change. First, the threshold $\underline{E}$ would decrease to a new threshold $\underline{E}'$ due to the reduction of total screening cost. The threshold E_1 would also decrease and get closer to $\underline{E}'$. $\overline{E}$ would increase to a new threshold $\overline{E}'$ because the marginal cost of screening decreases and the marginal benefit of screening is unchanged for a given bankruptcy exemption. Therefore, in equilibrium, banks continue screening till $\overline{E}'$. As a result, if the variable cost of screening becomes low, an increase in the bankruptcy exemption first leads to a decrease (increase) in the equilibrium loan size (gross interest rate) till $\underline{E}'$, and then leads to an increase (decrease) in the equilibrium loan size (gross interest rate) if the bankruptcy exemption continues increasing till $\overline{E}'$. Consequently, the relation between bankruptcy exemptions and loan size remains non-monotonic as found in Berkowitz and White (2004), but with the region with a positive slope becoming larger.

Finally, I consider the case in which banks' screening becomes more accurate. In particular, given the same screening intensity exerted, the banks can better distinguish the projects of good quality from the ones of bad quality, i.e., $\widetilde{\pi}(s) < \pi(s)$ for any $s > 0$. I conjecture that it has a similar effect as the reduction of per-unit cost of screening. Since now screening is more accurate a cretain screening outcome can be achieved at a lower screening intensity and thus at a lower cost. Hence, the threshold $\underline{E}$ and E_1 decrease. Besides, the same argument applies to an increase in threshold $\overline{E}$. As a result, if the screening accuracy improves considerably, the relation between bankruptcy exemptions and loan size is also non-monotonic, which is again the same as the relation found in Berkowitz and White (2004), and again the region with the positive slope becomes larger.

Our model conjectures that if there is a reduction in quasi-fixed cost or in variable cost of screening, or the improvement of screening accuracy, the relation between bankruptcy exemptions and small firms' loan size remains non-monotonic as found in Berkowitz and White (2004). It is reasonable to assume that in the last decades, the screening technology has been improved. In particular, if there is a considerable reduction in the quasi-fixed cost of screening together with a reduction in the per-unit screening cost, the relation between bankruptcy exemptions and small firms' loan size should become inverted U-shaped. Otherwise, the relation should remain the same non-monotonic relation similar with Berkowitz and White (2004). For testing the model, repeating the exercise of Berkowitz and White (2004) by using new waves of NSSBF data would be important and interesting. This is left for future research.

6 Conclusion

In this paper, I analyze the impact of bankruptcy exemptions on banks' lending by endogenizing banks' screening decisions, and then investigate its implication for aggregate production. I first build a principal-agent mode which concentrates on the incentive problem of banks. The model replicates the non-monotonic relation between bankruptcy exemptions and small firms' loan size, as well as gross interest rates, as observed in the data. In the model, I disentangle the total effect of bankruptcy exemptions into two effects: a negative *direct effect* on the loan size, and an positive *indirect effect* on the loan size which arises from banks' screening. Overall, the model shows that when bankruptcy exemptions are neither too low nor too high, the *indirect effect* dominates the *direct effect.* Therefore, the loan size increases as the bankruptcy exemption increases within this range. Otherwise, there is only the *direct effect*. As a result, the non-monotonic relation between bankruptcy exemptions and loan size as well as gross interest rates is established.

Furthermore, I show that, when incorporating banks' screening, in addition to the *intensive margin*, the *extensive margin* is an important channel for aggregate production: banks screen out bad-quality projects (although not perfectly), and fund good-quality projects. As a consequence, the set of funded projects contains a higher proportion of good-quality projects. Unlike stated in the previous literature, bankruptcy exemptions affect aggregate production non-monotonically. Specifically, banks' screening affects the allocation of credit between good-quality and bad-quality projects. Without banks' screening, banks provide credit to both. As bankruptcy exemptions increase, banks have less incentives to provide abundant and cheap credit. Consequently, aggregate production decreases due to fewer credit obtained (*intensive margin*). With banks' screening, banks can distinguish good-quality projects from bad-quality ones, although not perfectly. Therefore, credit is re-allocated more efficiently. In particular, credit flows from bad-quality projects to good-quality ones. This *extensive margin* of banks' lending decisions is crucial and is neglected in the previous literature.

More specifically, this paper shows that different levels of bankruptcy exemptions closely relate to banks' incentives to screen. When bankruptcy exemptions are very low and very high, banks do not screen at all. This leads to a decrease of aggregate expected production as the bankruptcy exemption increases. Interestingly, when bankruptcy exemptions are in a mid range, banks exercise more screening as the bankruptcy exemption rises. As a result, more credit flows from bad-quality projects to good-quality ones as the exemption increases within the mid range. As a consequence, the relation between bankruptcy exemptions and aggregate production is non-monotonic. The result implies that a high (low) bankruptcy exemption does not necessarily lead to a lower (higher) aggregate production.

Finally, I conjecture that if banks' screening improves due to a decrease in the variable cost of screening or an improvement of the screening accuracy, the relation between bankruptcy exemptions and small firms' loan size remains non-monotonic as in Berkowitz and White (2004), but with the two thresholds E_1 and $\underline{E}$ becoming smaller and getting closer to each other. If the screening improves due to a reduction in the quasi-fixed cost, the relation is non-monotonic as well, but with the threshold $\underline{E}$ becoming small. In sum, my result suggests that banks' screening plays an important role in banks' lending decisions. In particular, banks' screening improves the efficiency in allocating credit to small firms. These testable implications are left for future research.

7 Reference

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8 Appendix

Proof. Lemma 1

Banks compete with other banks and choose screening intensity by taking other banks' contracts and other banks' screening intensity as given. Suppose that a debt contract $\left(B^{'}, r^{'}|A^{*}\right)$ given s^{*} is offered in equilibrium, and the bank earns a positive profits. However, another bank would offer another debt contract $(B'', r''|A^{*})$ given s^{*} such that $B' = B''$, but $r'' < r'$ but the bank still earns positive profits. Hence, it is clear that the bank which offers $(B', r'|A^{*})$ would not be able to attract any firms. By doing so, in equilibrium, given s^{*}, banks earn zero profit. ■

Proof. Proposition 2

Suppose that in equilibrium, $s^{*} > 0$, I can derive the equilibrium contracts $(B, r|A^{*})$ which make the banks earn zero profit

$$p(B)Br + (1 - p(B))(A^{*} - E) - B + \pi(s^{*})(A^{*} - E - B) = 2C(s)$$

However, if these contracts are offered in equilibrium, another bank can offer exactly the same contract with $s^{*} = 0$ such that the bank earns positive profits for some E.

Therefore, in equilibrium, $s^{*} > 0$ if and only if

$$2C(s^{*}) < (1 - \pi(s^{*}))(B - A^{*} + E)$$

which is equivalent to

$$\frac{2C(s^{*})}{1 - \pi(s^{*})} - B + A^{*} < E \tag{1}$$

where s^{*} satisfies

$$\pi(s^{*}) = \frac{2c}{B - A^{*} + E} = e^{-s^{*}} \tag{2}$$

we substitute s^{*} into equation (1), and we have

$$2c_f + 2c\ln(B - A^{*} + E) - 2c\ln 2c - B + A^{*} + 2c < E$$

Hence, we can find a threshold $E = \underline{E}$ such that $2c_f + 2c\ln(B - A^{*} + \underline{E}) - 2c\ln 2c - B + A^{*} + 2c = \underline{E}$. For $E \in [0, \underline{E}]$, the bank can earn positive profit with $s^{*} = 0$. Therefore, we show that in equilibrium, $s^{*} = 0$ for $E \in [0, \underline{E}]$.

Moreover, for $E \in \left[\overline{E}, \infty\right]$, in the following, we show that if $s^* > 0$ in equilibrium, $EU < V(A^*)$. This implies that firms will not participate in the credit market. $EU < V(A)$ if and only if

$$\begin{aligned} & p(B)\theta - (1+\pi(s^*))B \\ < \; & -(1-p(B)+\pi(s^*))(A^*-E) + (2-p(B))(V(A^*)-V(E)) + 2C(s^*) \end{aligned} \quad (3)$$

We substitute (2) into (3), and we can find a threshold $E = \overline{E}$ such that

$$\begin{aligned} & p(B)\theta - \left(1+\frac{2c}{B-A^*+\overline{E}}\right)B \\ = \; & -\left(1-p(B)+\frac{2c}{B-A^*+\overline{E}}\right)\left(A^*-\overline{E}\right) + (2-p(B))\left(V(A^*)-V\left(\overline{E}\right)\right) \\ & +2c_f + 2c\ln\left(B-A^*+\overline{E}\right) - 2c\ln 2c \end{aligned}$$

Moreover, we show that

$$p(B)\theta - 2B \geq -(2-p(B))(A^*-E) + (2-p(B))(V(A^*)-V(E)) \quad \text{for all } E \in \left[\overline{E}, \infty\right]$$

■

Proof. Lemma 3

$$\frac{\partial B}{\partial E}|_{s=s'} = \frac{-\frac{\partial^2 EU}{\partial B \partial E}}{\frac{\partial^2 EU}{\partial B^2}} = \frac{p'(B)(1-V'(E))}{-p''(B)(\theta+V(A^*)-A^*+E-V(E))} < 0$$

since $1 - V(E) < 0$ and $p''(B) < 0$ ■

Proof. Lemma 4

We have

$$\frac{\partial B}{\partial s^*} = \frac{-\frac{\partial^2 EU}{\partial B \partial s}}{\frac{\partial^2 EU}{\partial B^2}}|_{s=s^*} = \frac{-\frac{2c}{B-A^*+E}}{p''(B)(\theta+V(A^*)-A^*+E-V(E))} > 0$$

and

$$\frac{\partial s^*}{\partial E} = \frac{-\frac{\partial^2 EU}{\partial s \partial E}}{\frac{\partial^2 EU}{\partial s^2}}|_{s=s*} = \frac{1}{B-A^*+E} > 0$$

Therefore, $\frac{\partial B}{\partial s^*}\frac{\partial s^*}{\partial E} > 0$ ■

Proof. Proposition 5

For $E \in [0, \underline{E}]$ and $E \in [\overline{E}, \infty)$, $s^* = 0$ is shown is Proposition 2. Therefore, the equilibrium contracts solve the following problem:

$$\max_{B,r} EU = \frac{1}{2}\left(p(B)\left(\theta - Br + V(A^*)\right) + (1 - p(B))\left(V(E)\right)\right) + \frac{1}{2}V(E)$$

s.t.

$$E\pi = \frac{1}{2}\left(p(B)Br + (1 - p(B))(A^* - E) - B\right) + \frac{1}{2}(A^* - E - B) \geq 0 \quad (\mathrm{PC}_B)$$

From Lemma 1, we know that banks' participation constraint binds in equilibrium. Therefore, we substitute r from the banks' participation constraint into the objective function. By taking the first derivative with respect to B, we have

$$p'(B)\left(\theta + V(A^*) - A^* + E - V(E)\right) - 2 = 0 \tag{4}$$

The equilibrium loan size with screening must satisfy the above equation.

By Envelope Theorem, we derive

$$\frac{\partial B^*}{\partial E} = -\frac{\frac{\partial^2 EU}{\partial B \partial E}}{\frac{\partial^2 EU}{\partial B^2}} = \frac{p'(B^*)\left(1 - V''(E)\right)}{-p''(B^*)\left(\theta + V(A^*) - A^* + E - V(E)\right)} < 0$$

since $V'(E) > 1$ and $p''(B^*) < 0$. ■

Proof. Proposition 6

Equilibrium contracts must be the solution to the following problem,

$$\max_{B,r,s} EU = \frac{1}{2}\left(p(B)\left(\theta - Br + V(A^*)\right) + (1 - p(B))\left(V(E)\right)\right) + \frac{1}{2}V(E)$$

s.t.

$$E\pi = \frac{1}{2}\left(p(B)Br + (1 - p(B))(A^* - E) - B\right) + \frac{1}{2}\pi(s)(A^* - E - B) - C(s) \geq 0$$

$$EU = \frac{1}{2}\left(p(B)\left(\theta - Br + V(A^*)\right) + (1 - p(B))\left(V(E)\right)\right) + \frac{1}{2}V(E) \geq V(A^*)$$

Equilibrium B and s must satisfy:

F.O.C of B

$$p'(B)\left(\theta + V(A^*) - A^* + E - V(E)\right) - (1 + \pi(s)) = 0 \tag{1}$$

F.O.C of s

$$-2c - \pi'(s)(B - A^* + E) = 0 \tag{2}$$

From Lemma 3 and 4, we obtain both Direct Effect and Indirect Effect. And we have show that

$$\underbrace{|p'(B)(1 - V'(E))|}_{(I)} \geq \underbrace{\frac{2c}{(B - A^* + E)^2}}_{(II)} \quad \forall \underline{E} < E \leq E_1 \tag{3}$$

and

$$|p'(B)(1 - V'(E))| < \frac{2c}{(B - A^* + E)^2} \quad \forall E_1 < E < \overline{E} \tag{4}$$

Therefore, we have to show $|p'(B)(1 - V(E))|$ and $\frac{2c}{(B-A+E)^2}$ cross only once at $E = E_1$ and moreover, equation (3) and (4) are satisfied.

Term (I) and term (II) are both decreasing in E, and we have to show that

$$\lim_{E \to \underline{E}} |p'(B)(1 - V'(E))| > \lim_{E \to \underline{E}} \frac{2c}{(B - A^* + E)^2}$$

and

$$\lim_{E \to \overline{E}} |p'(B)(1 - V'(E))| < \lim_{E \to \overline{E}} \frac{2c}{(B - A^* + E)^2}$$

Moreover, we know that since when $E \to \underline{E}$ and when $E \to \overline{E}$, equilibrium $s^* = 0$ form proposition 2. Hence, we have

$$\lim_{E \to \underline{E}} \frac{2c}{(B - A + E)^2} = \frac{1}{B - A^* + \underline{E}} = \frac{1}{2c}$$

and

$$\lim_{E \to \overline{E}} \frac{2c}{(B - A + E)^2} = \frac{1}{B - A^* + \overline{E}} = \frac{1}{2c}$$

Hence, we have to show that

$$2(V'(\underline{E}) - 1) > \frac{1}{2c}$$

and

$$2\left(V'(\overline{E}) - 1\right) < \frac{1}{2c}$$

And we already know that $2\left(V'\left(\underline{E}\right)-1\right) > 2\left(V'\left(\overline{E}\right)-1\right)$ since $V(E)$ is an increasing and concave function of E. As long as there exists a $c > 0$ such that

$$2\left(V'\left(\underline{E}\right)-1\right) > \frac{1}{2c} > 2\left(V'\left(\overline{E}\right)-1\right)$$

We can show that there also exists a threshold $E = E_1 \in \left[\underline{E}, \overline{E}\right]$ such that $2\left(V'\left(E_1\right)-1\right) = \frac{1}{2c}$, and equation (3) and (4) are satisfied. ■

Proof. Proposition 7

This result simply comes from combining the two results of Proposition 5 and 6. ■

Proof. Proposition 8

First, it is easy to show that if banks exert positive screening intensity $s^* > 0$ (when $E \in \left[\underline{E}, \overline{E}\right]$), in equilibrium, $B^* r^*$ is an increasing and concave function of B^*. Hence, $\frac{\partial(B^* r^*)}{\partial B^*} < r^*$ due to the concavity of $B^* r^*$. Therefore, the effect of bankruptcy exemptions on the equilibrium gross interest rate is the following:

$$\begin{aligned}
& \frac{\partial r^*}{\partial E} \\
= & \frac{\partial\left(B^* r^*\right)\left(B^*\right)^{-1}}{\partial E} \\
= & \left(B^*\right)^{-1} \underbrace{\frac{\partial\left(B^* r^*\right)}{\partial B^*}}_{=0} \frac{\partial B^*}{\partial E} + B^* r^* (-1) \left(B^*\right)^{-2} \frac{\partial B^*}{\partial E} \\
= & \underbrace{\left(\frac{1}{B^*} \frac{\partial\left(B^* r^*\right)}{\partial B^*} - \frac{r^*}{B^*}\right)}_{-} \frac{\partial B^*}{\partial E}
\end{aligned}$$

Therefore, from Proposition 6, we can show that $\frac{\partial r^*}{\partial E} > 0$ if $E \in \left[\underline{E}, E_1\right]$ and $\frac{\partial r^*}{\partial E} < 0$ if $E \in \left[E_1, \overline{E}\right]$.

Second, if banks do not screening $s^* = 0$ (when $E \in \left[0, \underline{E}\right]$ and $E \in$

$\left[\overline{E},\infty\right)$),

$$
\begin{aligned}
& \frac{\partial r^*}{\partial E} \\
= \; & \frac{\partial \left(B^* r^*\right)\left(B^*\right)^{-1}}{\partial E} \\
= \; & \left(B^*\right)^{-1} \frac{\partial \left(B^* r^*\right)}{\partial B^*} \frac{\partial B^*}{\partial E} + B^* r^* \left(-1\right)\left(B^*\right)^{-2} \frac{\partial B^*}{\partial E} \\
= \; & \left(\frac{1}{B^*} \frac{\partial \left(B^* r^*\right)}{\partial B^*} - \frac{r^*}{B^*}\right) \underbrace{\frac{\partial B^*}{\partial E}}_{-}
\end{aligned}
$$

As long as $\frac{\partial(B^* r^*)}{\partial B^*} < r^*$, $\frac{\partial r^*}{\partial E} > 0$.

$$\frac{\partial \left(B^* r^*\right)}{\partial B^*} = \frac{2p\left(B^*\right) - 2p'\left(B^*\right) B^*}{p\left(B^*\right)^2} < \frac{\left(B^* - A^* + E\right) + B^* - \left(1 - p\left(B^*\right)\right)\left(A^* - E\right)}{p\left(B^*\right) B^*} = r^*$$

which is equivalently to

$$\underbrace{\frac{B^*}{p\left(B^*\right)\left(2 - p\left(B^*\right)\right)\left(\theta + V\left(A^*\right) - A^* + E - V\left(E\right)\right)}}_{(1)} > \underbrace{\frac{A^*}{B^* - A^*}}_{(2)} \qquad \text{(i)}$$

Term (1) is an increasing function of B^* and term (2) is a decreasing function of B^*. Moreover,

$$\lim_{B^* \to 0} \frac{B^*}{p\left(B^*\right)\left(2 - p\left(B^*\right)\right)\left(\theta + V\left(A\right) - A + E - V\left(E\right)\right)} > 0 > \lim_{B^* \to 0} \frac{A}{B^* - A} = -1$$

. Therefore, equation (i) holds for all B^*. As a result, $\frac{\partial(B^* r^*)}{\partial B^*} < r^*$ and thus, $\frac{\partial r^*}{\partial E} > 0$ for $E \in [0, \underline{E}]$ and $E \in \left[\overline{E}, \infty\right)$. ■

Proof. Proposition 9

We take derivative of $\frac{1}{1+\pi(s^+)} p\left(B^*\right)$ with respect to E. It can be written as

$$\frac{\partial \left(\frac{1}{1+\pi(s^+)} p\left(B^*\right)\right)}{\partial E} = \frac{p'\left(B^*\right) \frac{\partial B^*}{\partial E}\left(1 + \pi\left(s^*\right)\right) - p\left(B^*\right) \pi'\left(s^*\right) \frac{\partial s^*}{\partial E}}{\left(1 + \pi\left(s^*\right)\right)^2}$$

Since we know from Lemma 3, 4 and Proposition 6, for $E \in \left(E_1, \overline{E}\right)$, $p'\left(B^*\right) > 0$ and $\frac{\partial B^*}{\partial E} > 0$, and given we know that $\pi'\left(s^*\right) < 0$ and $\frac{\partial s^*}{\partial E} > 0$, it is shown that when $E \in \left(E_1, \overline{E}\right)$, $\frac{\partial\left(\frac{1}{1+\pi(s^+)}p(B^*)\right)}{\partial E} > 0$. As for $E \in [\underline{E}, E_1]$, $p'(B^*) > 0$ and $\frac{\partial B^*}{\partial E} < 0$. Moreover, given we know $\pi'(s^*) < 0$ and $\frac{\partial s^*}{\partial E} > 0$. Thus

$$\lim_{E \to E_1} \frac{\partial\left(\frac{1}{1+\pi(s^+)}p(B^*)\right)}{\partial E} = \frac{p'(B^*)\frac{\partial B^*}{\partial E_1}(1+\pi(s^*)) - p(B^*)\pi'(s^*)\frac{\partial s^*}{\partial E_1}}{(1+\pi(s^*))^2}$$

Since $\frac{\partial B^*}{\partial E_1} = 0$, we have

$$\lim_{E \to E_1} \frac{\partial\left(\frac{1}{1+\pi(s^+)}p(B^*)\right)}{\partial E} = \frac{-p(B^*)\pi'(s^*)\frac{\partial s^*}{\partial E_1}}{(1+\pi(s^*))^2} > 0$$

This implies that there exist another threshold E_2 and $E_2 < E_1$ such that $\lim_{E \to E_2} \frac{\partial\left(\frac{1}{1+\pi(s^+)}p(B^*)\right)}{\partial E} = 0$. Therefore, for any $E \in \left[E_2, \overline{E}\right]$, $\frac{\partial\left(\frac{1}{1+\pi(s^+)}p(B^*)\right)}{\partial E} \geq 0$
■